Loss-Tolerant Distributed Lattice Surgery using Fusion Networks

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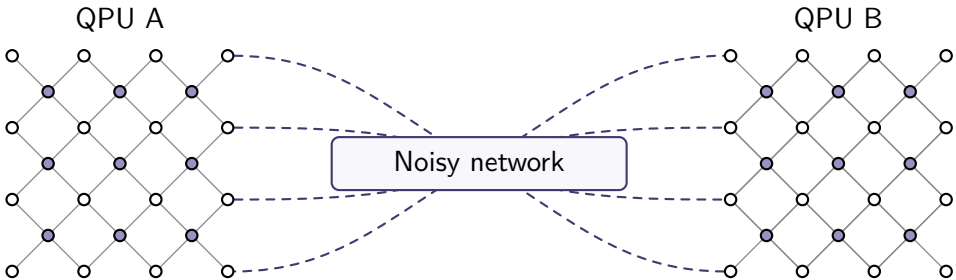


arXiv:2610.01923

Introduction

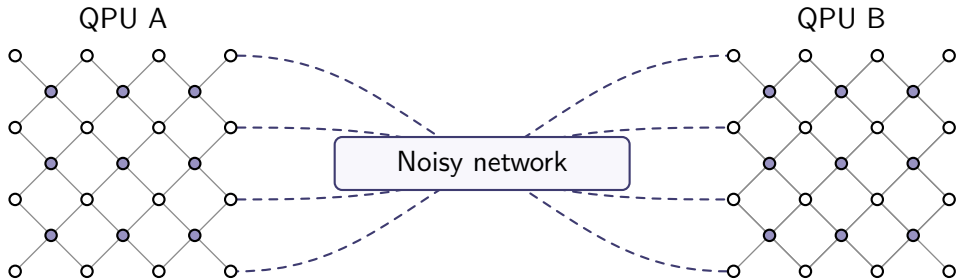
Introduction

- Reaching practical scale fault-tolerant quantum computing is likely to require a network of interconnected quantum processing units (QPUs).
- Stationary, matter-based qubits for processing and flying, photonic qubits for communication.



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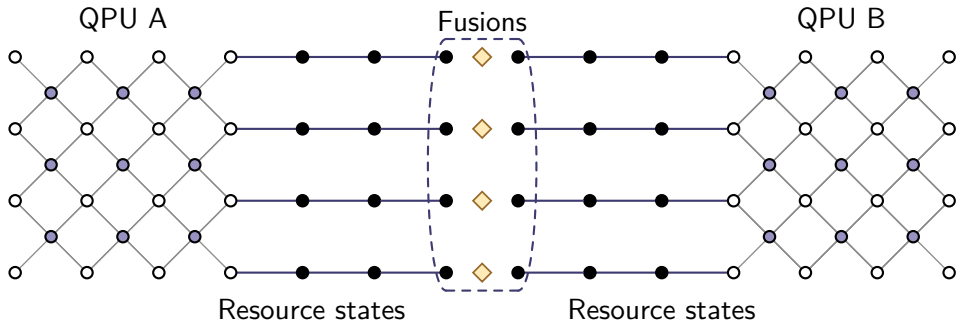
"Black-box" source of noisy entanglement – error correction must tolerate.

The network as a photonic processor

In photonic quantum computing, the network *is* the computer.

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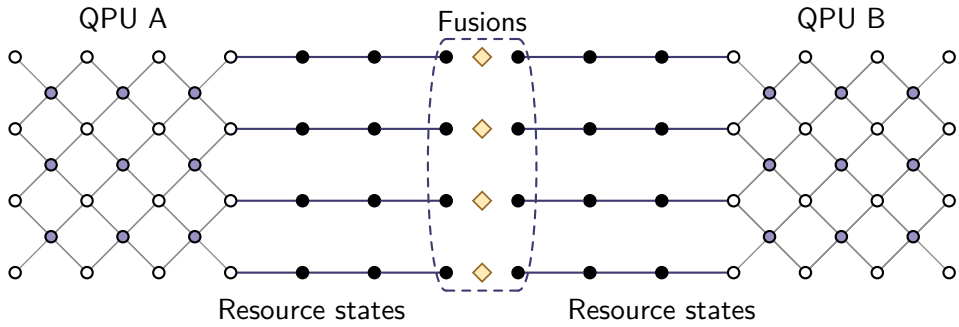


Error correction performed using resource states and fusions[1].

[1] Bartolucci et al., "Fusion-Based Quantum Computation".

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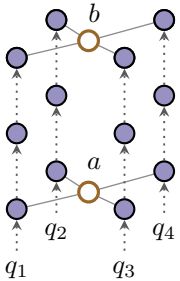
Error correction performed using resource states and fusions[1].
Can we exploit this in a DQC setting?

[1] Bartolucci et al., "Fusion-Based Quantum Computation".

From circuits to clusters and fusion networks

Circuit- and measurement-based error correction

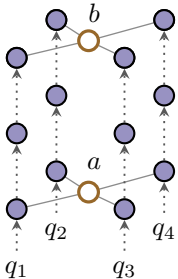
- Circuit-based QEC: parity checks from stabilisers measured over time.



$$a \oplus b = 0$$

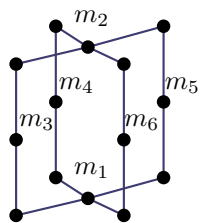
Circuit- and measurement-based error correction

- Circuit-based QEC: parity checks from stabilisers measured over time.



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- Measurement-based QEC: checks from resource state stabilisers and measurements[2].



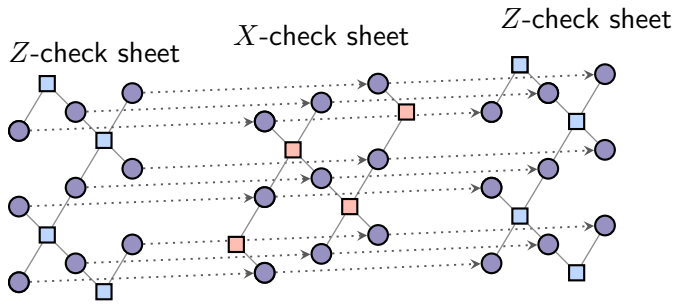
$$|G\rangle = \prod_{(u,v)\in E} CZ_{uv} \, |+\rangle^{\otimes V}$$

$$m_1 \oplus \cdots \oplus m_6 = 0$$

[2]Raussendorf, Harrington, and Goyal, "A Fault-Tolerant One-Way Quantum Computer".

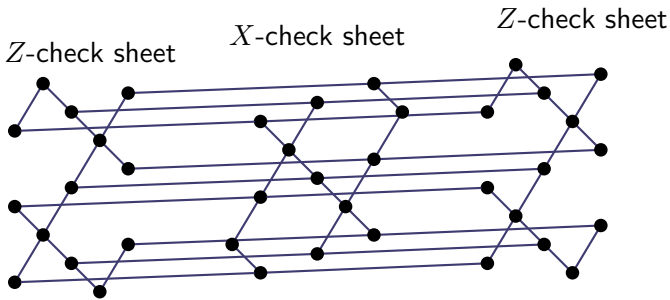
Foliated error correction codes

Circuit-based error correction stacks stabiliser measurement layers in *time*.



Foliated error correction codes

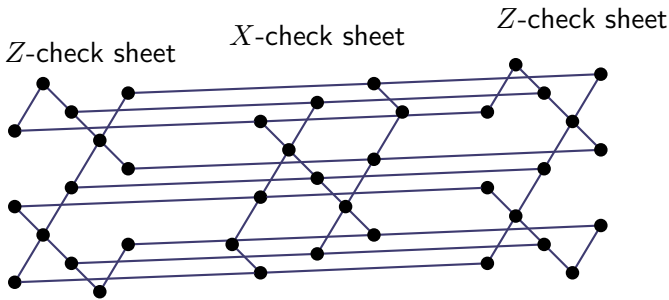
A "foliated" error correction code stacks resource states in layers in *space*[3].



[3] Bolt et al., "Foliated Quantum Error-Correcting Codes".

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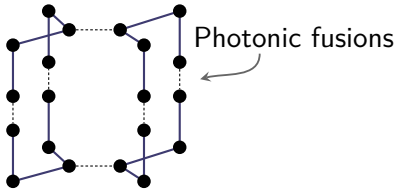
Circuit-based codes and foliated codes share the same **detecting regions** and **logicals**[4].

[3] Bolt et al., "Foliated Quantum Error-Correcting Codes".

[4] Bombin et al., "Unifying Flavors of Fault Tolerance with the ZX Calculus".

From a cluster to a fusion network

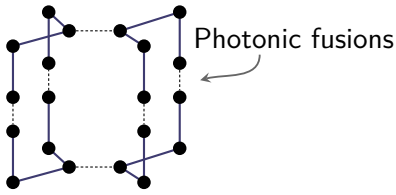
Cluster states can be decomposed into smaller resource states connected by *fusion measurements*[5].



[5] Bartolucci et al., "Fusion-Based Quantum Computation".

From a cluster to a fusion network

Cluster states can be decomposed into smaller resource states connected by *fusion measurements*[5].



Fusions measure joint parities – cluster preparation and measurement in one step. Loss and fusion failure must be *decoded*.

[5] Bartolucci et al., "Fusion-Based Quantum Computation".

Hybrid syndrome extraction

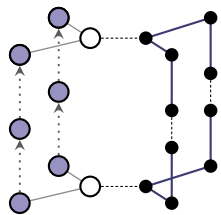
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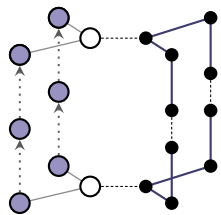
Equivalent detecting regions can be generated using a combination of time-evolving qubits, resource states, and fusions.



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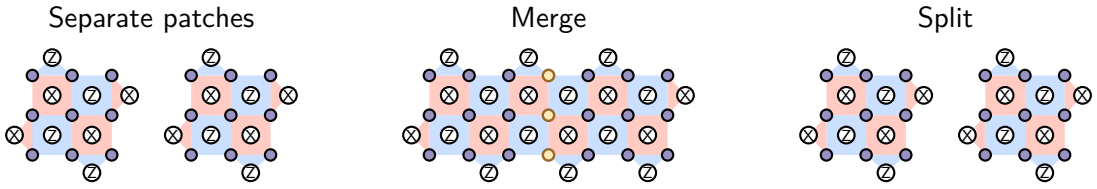


Can we use this to construct hybrid logical circuits for matter-photonic systems?

Hybrid lattice surgery

The lattice-surgery construction

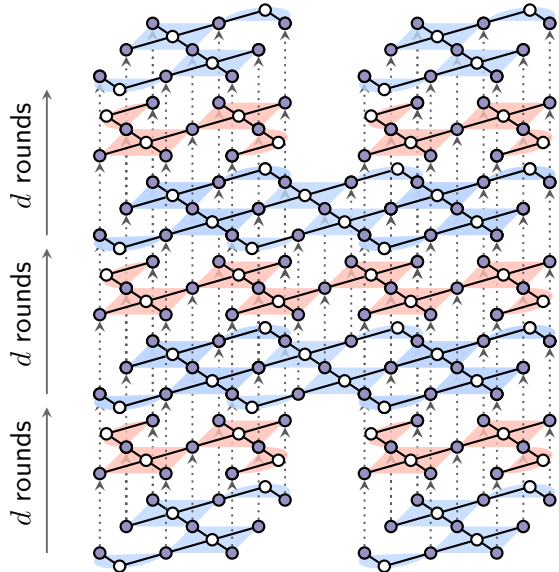
A ZZ merge on two $|+_L\rangle$ patches prepares a logical Bell state[6].



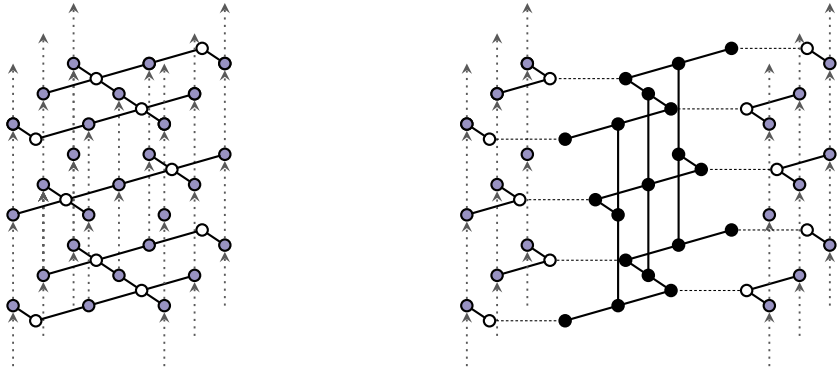
- Prepare the seam in $|+\rangle$, measure joint checks, then split by X readout.
- Projectively measure the merge observable $\bar{Z}\bar{Z}$ and protect the perpendicular correlation $\bar{X}\bar{X}$.

[6] Horsman et al., "Surface Code Quantum Computing by Lattice Surgery".

Space-time diagram

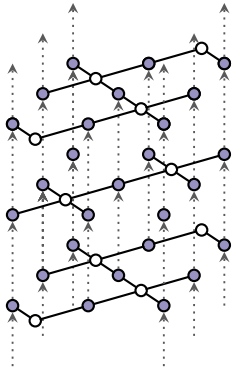


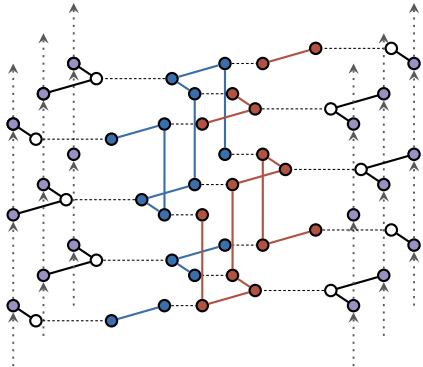
Hybridising the surgery interface



- Local circuit QEC couples to a foliated seam via boundary fusions.

Hybridising the surgery interface

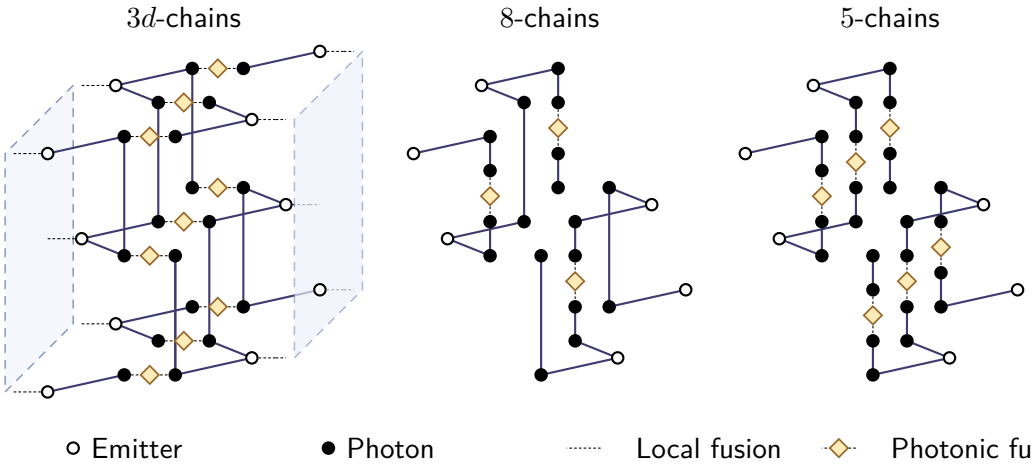




- Local circuit QEC couples to a foliated seam via boundary fusions.
- Cluster-state interface can be decomposed into smaller resource states.

Resource-state decompositions

The interface can be decomposed into smaller emitter/photon chains.

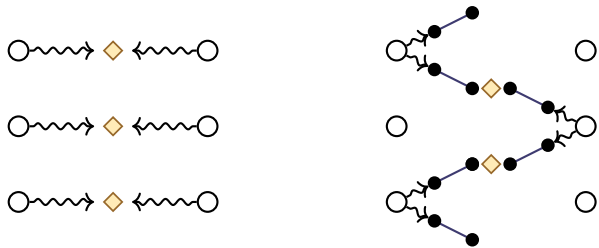


Interface detector error models

- Network noise is dominated by photon loss and probabilistic fusions causing *erasures*.
- Erasure tolerance can be analysed by the interface-only noise model.

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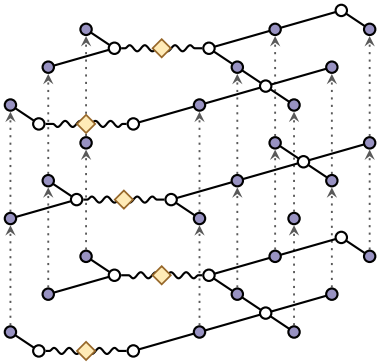


- We can treat both hybrid interface and Bell-pair interface errors under this model.

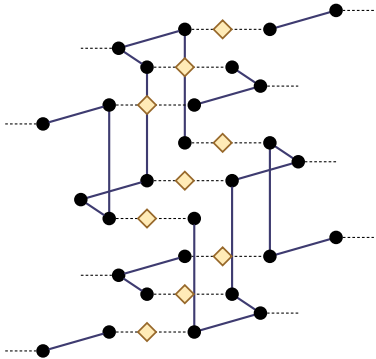
Interface detector error models

Hybrid surgery modifies the interface geometry relative to a Bell-pair interface.

Bell interface

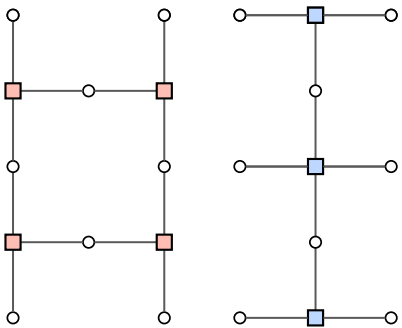


Hybrid interface

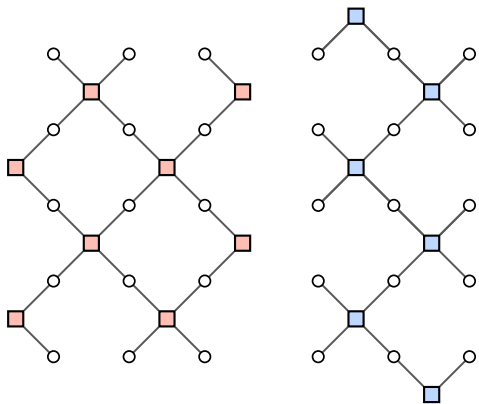


Interface detector error models

Unrotated surface code geometry

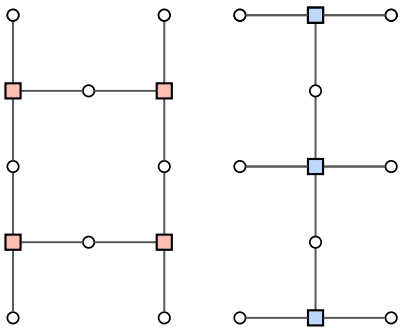


Rotated surface code geometry

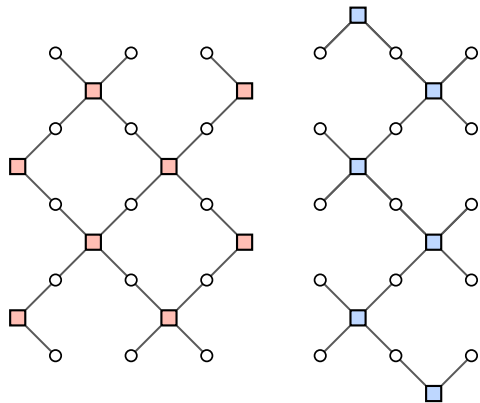


Interface detector error models

Unrotated surface code geometry



Rotated surface code geometry



Hybrid interface doubles distance on both $\bar{Z}\bar{Z}$ and $\bar{X}\bar{X}$ observables via rotated geometry!

Interface distances and bulk thresholds

Erasure thresholds at interface can be predicted using percolation theory[7].

Protocol	$d_{\text{int}}(\bar{Z}\bar{Z})$	$d_{\text{int}}(\bar{X}\bar{X})$	p_e^*
Bell	$d+1$	$\lfloor (d+1)/2 \rfloor$	50%
BellZigZag[8]	$d+1$	d	50%
$3d$-Chain	$2d+1$	d	50%
8-Chain	$2d+1$	d	38.2%
5-Chain	$2d+1$	d	31.8%

- Distances count only interface outcomes – full distance also depends on the local implementation and noise model.
- Truncation preserves distance but lowers thresholds.

[7] Stace, Barrett, and Doherty, "Thresholds for Topological Codes in the Presence of Loss".
[8] Jacinto, Gouzien, and Sangouard, *Network Requirements for Distributed Quantum Computation*.

Numerical results

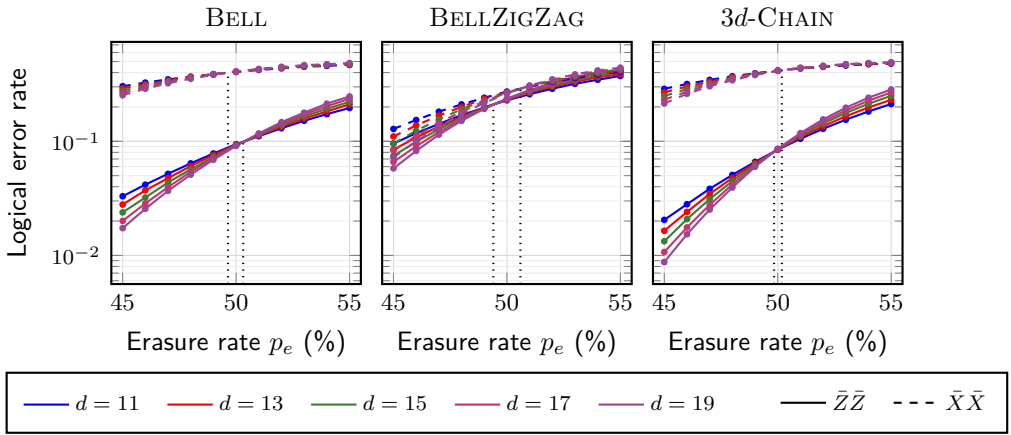
Simulation and noise models

- Stim circuits, correlated MWPM in PyMatching 2, and per-shot erasure reweighting.[9]

Noise	Operations	Parameter
Local circuit	Two-qubit depolarisation, reset/readout flips	p_c
Resource state	Emitter reset and fusion-outcome flips	p_r
Network erasure	Heralded elementary fusion-outcome erasure	p_e

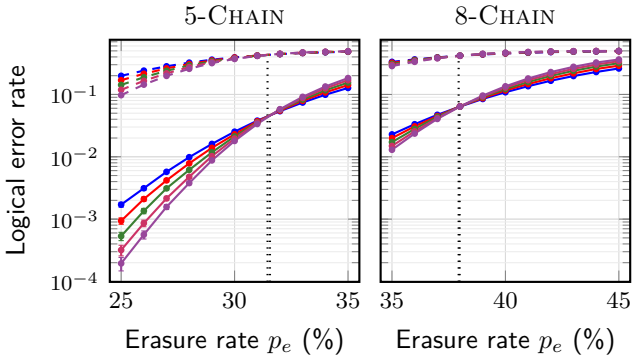
[9] Gidney, "Stim"; Higgott and Gidney, "Sparse Blossom".

Pure-erasure thresholds



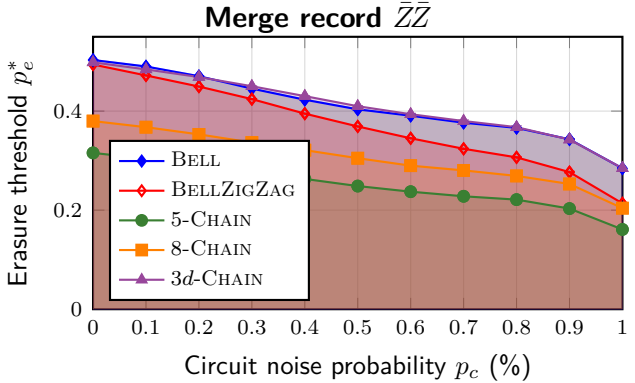
$p_c = p_r = 0$. Bulk prediction: 50%.

Pure-erasure thresholds



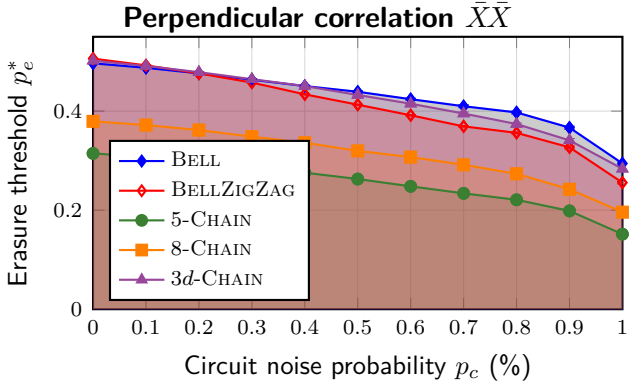
$p_c = p_r = 0$. Bulk predictions: 31.8% and 38.2%.

Correctable regions with local circuit noise



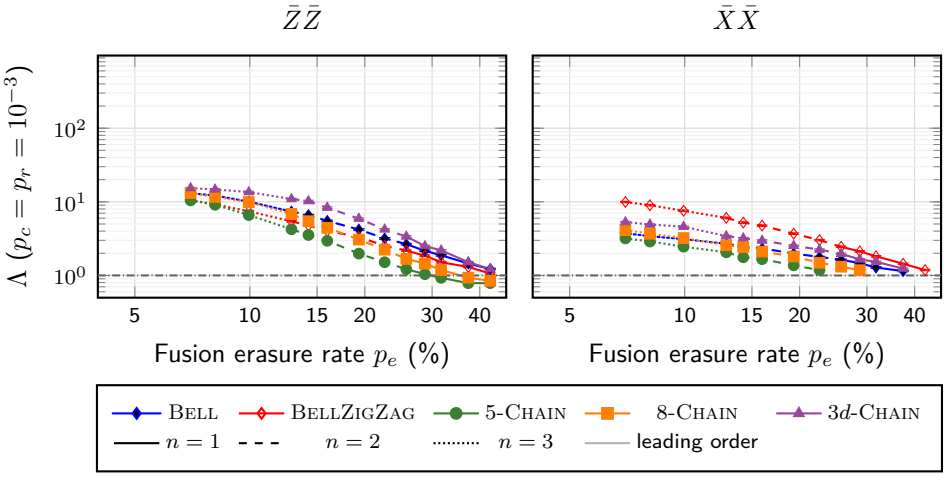
- $p_r = 0$. Circuit noise lowers every erasure threshold, with steeper degradation at larger p_c.

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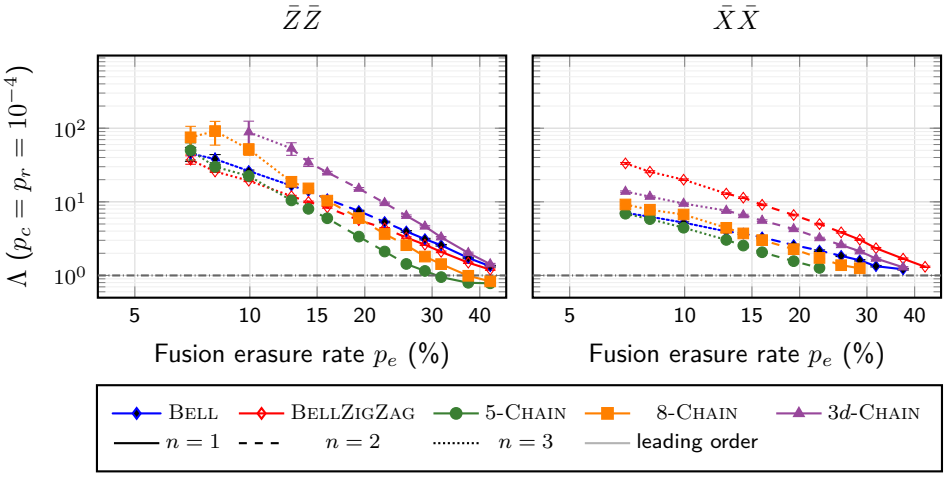
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Subthreshold logical suppression



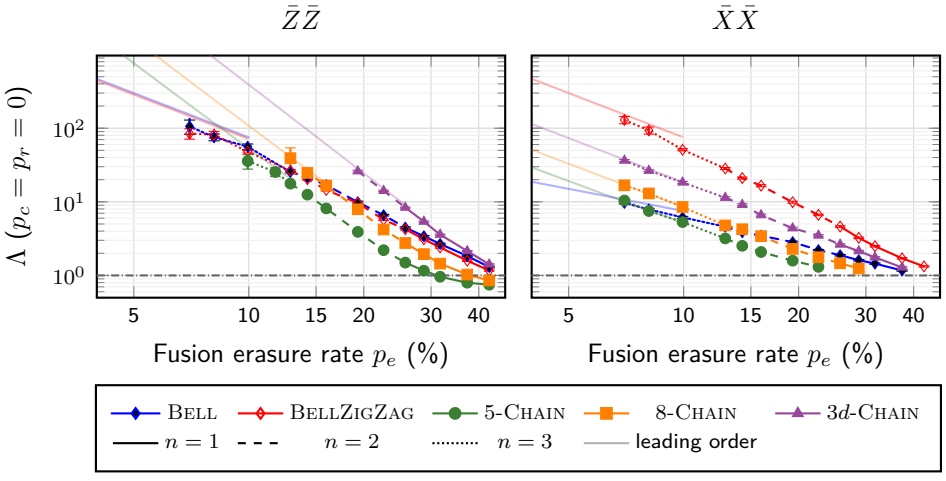
$\Lambda_{5\to7} = p_L(5)/p_L(7)$ at $p_c = p_r = 10^{-3}$.

Subthreshold logical suppression



$\Lambda_{5\to7} = p_L(5)/p_L(7)$ at $p_c = p_r = 10^{-4}$.

Subthreshold logical suppression



$\Lambda_{5\to7} = p_L(5)/p_L(7)$ at $p_c = p_r = 0$.

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Read the full paper for details: arXiv:2610.01923

Thank you!

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Syndrome extraction and its realisations

- QEC protocols can share one **syndrome extraction diagram (SED)**.[10]
- Hybrid realisations assign different regions to operations suited to their hardware.

Level	What it specifies
SED	Detecting regions and boundary correlations
Realisation	Qubit world-lines and classical outcomes
Hardware mapping	Physical operations and their noise

[10] Bombin et al., "Unifying Flavors of Fault Tolerance with the ZX Calculus".

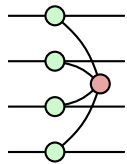
Building a syndrome extraction diagram

For each CSS generator S, a phase-free ZX gadget represents

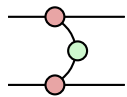
$$\Pi_S = \frac{I + S}{2}.$$

- Compose alternating Z- and X-type generator layers.
- Fuse neighbouring spiders of the same colour to obtain the SED.
- Qubit identity and classical outcomes enter with the realisation.

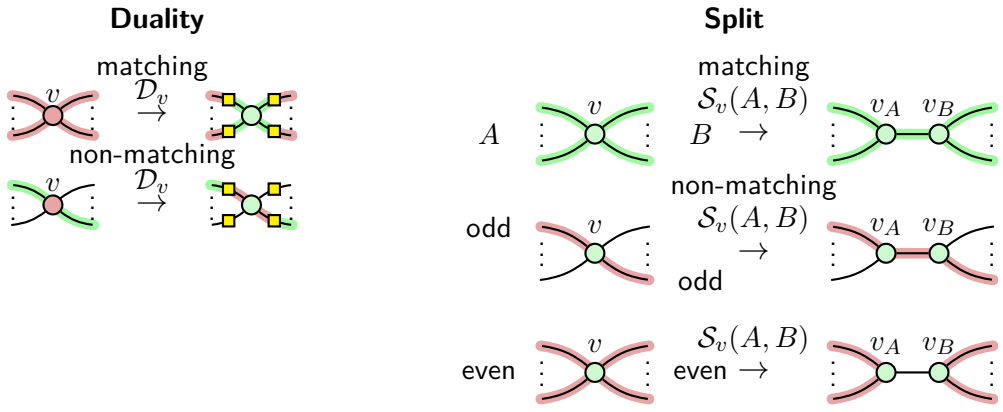
Π_{ZZZZ}



Π_{XX}



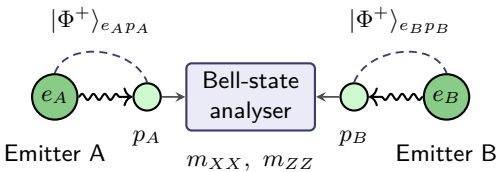
Web-preserving transformations



- A web bijection retains the detecting regions and quantum boundary correlations.
- Fault distance also depends on the realisation and noise model.

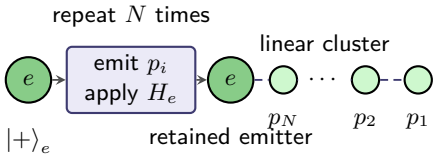
Emitter mapping of resource-state realisations

Remote Bell pairs



- Fuse photons emitted by two matter qubits.
- Retain the emitters for local interactions and readout.

Linear resource states



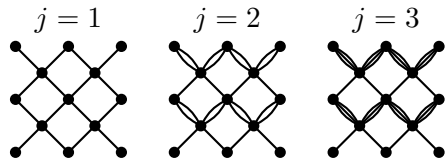
- Repeated emission and rotation generate a chain.[a]
- Matter and photonic qubits can belong to the same resource state.

[a] Lindner and Rudolph, "Proposal for Pulsed On-Demand Sources of Photonic Cluster State Strings".

Truncation and the erasure threshold

- Subdivision adds contributions with the same detector and logical labels.
- Connectivity and interface distance stay unchanged.
- A bond with j independent contributions has erasure probability

$$P_j = 1 - (1 - p_e)^j.$$

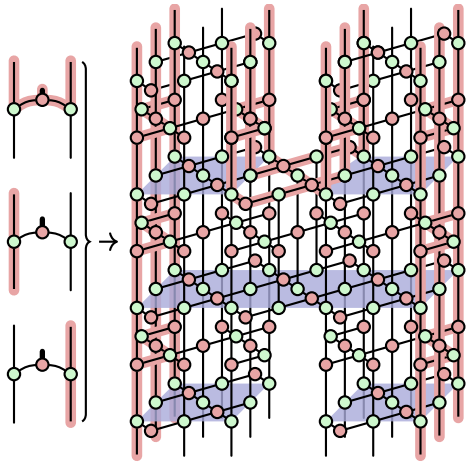


The self-dual bulk condition gives[a]

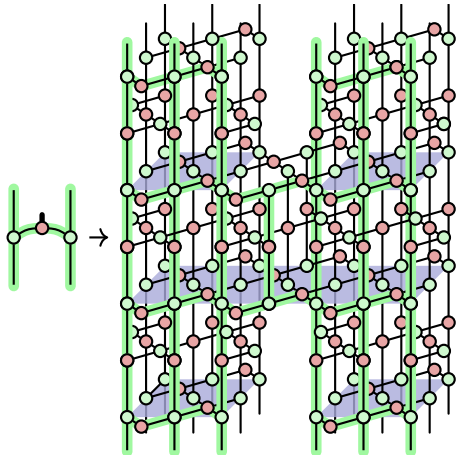
$$P_j + P_1 = 1 \quad \Longrightarrow \quad p_e^* = (1 - p_e^*)^j.$$

[a] Wu, "Critical Point of Planar Potts Models".

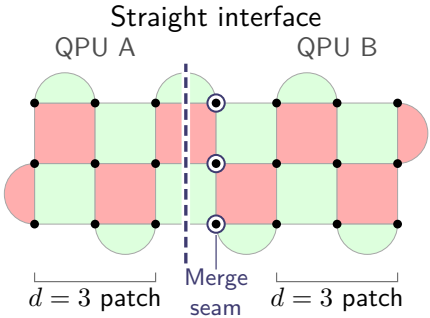
Merge-record correlation in the surgery SED



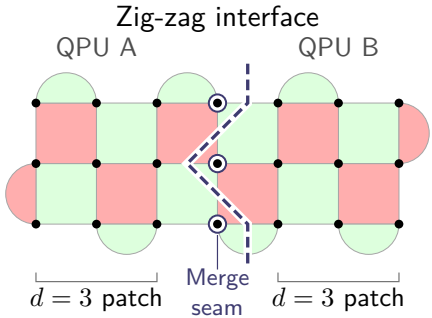
Perpendicular correlation in the surgery SED



Straight and zig-zag Bell interfaces



Crossing bulk stabilisers: $2 + 2$

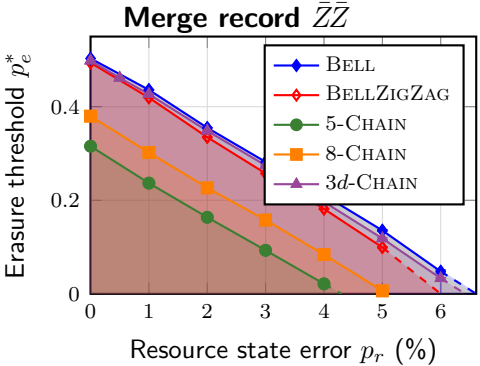


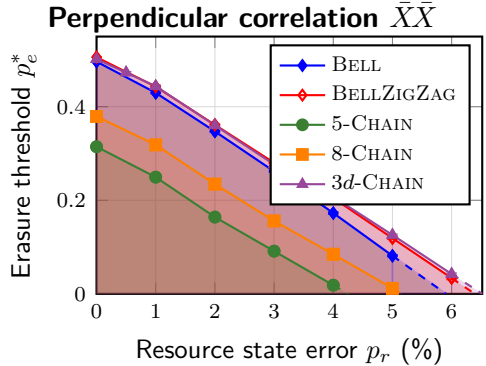
Crossing bulk stabilisers: $3 + 1$

• Data qubit ◉ Seam qubit Z stabiliser X stabiliser QPU partition

The $3 + 1$ partition restores $d_{\text{int}}(\bar{X}\bar{X}) = d$ without changing the square-lattice bulk threshold.

Correctable regions with resource-state noise





- $p_c = 0$. Resource-state errors lower the erasure thresholds approximately linearly.
- Fusion-outcome flips are unheralded, in addition to the heralded erasures.

Photon loss and fusion boosting

$$p_e = 1 - (1 - 2^{-(n+1)})(1 - p_l)^{2^n}, \qquad p_l^* = 1 - \left(\frac{1 - p_e^*}{1 - 2^{-(n+1)}}\right)^{2^{-n}}.$$

- p_l is photon-loss probability. Boosting uses $2^n - 2$ ancillary photons.[11]
- Unboosted fusion ($n = 1$) already gives $p_e = 25\%$ at zero photon loss.
- More boosting reduces fusion failure but increases exposure to photon loss.
- In pure erasure, $p_e^* = 50\%$ gives $p_l^* = 18.4\%$ at $n = 1$.
- At $p_c = 1\%$, the available merge estimates favour $n = 2$ for Bell, BellZigZag and $3d$-Chain, and $n = 3$ for 5-Chain.

[11] Bartolucci et al., "Fusion-Based Quantum Computation"; Grice, "Arbitrarily Complete Bell-state Measurement Using Only Linear Optical Elements".

Leading-order erasure estimates

Under pure erasure, a logical-supporting erased set causes failure with probability $1/2$.

$$p_L(d) \approx \tfrac{1}{2}\mathcal{N}_d p_e^{w_d}, \qquad \Lambda_{5\to 7} \approx \frac{\mathcal{N}_5}{\mathcal{N}_7} p_e^{-(w_7 - w_5)}.$$

- w_d is the minimum logical weight. $\mathcal{N}_d$ counts minimum-weight logical supports in the interface DEM.
- For the primal DEM at $d = 7$, $3d$-Chain has about 4.6×10^4 such supports, compared with four for Bell.
- Merge suppression scales as p_e^{-4} for chains, against p_e^{-2} for Bell. The larger chain prefactor delays the crossover.
- Low logical error rates make direct Monte Carlo estimation difficult in this regime.

Local fault mechanisms and scope

- Deterministic boundary Hadamard fusions can introduce half-distance fault chains on the perpendicular observable.
- Flagged gadgets or alternative distance-preserving boundary implementations could address this limitation.
- Simulations assume deterministic supply of resource states and a simplified preparation-noise model.
- Interface distance and suppression gains do not establish lower emitter count, preparation time or total overhead.
- Comparing erasure decoding with post-selection needs explicit hardware and rate assumptions.